Non-Parametric Bayesian Dictionary Learning for Sparse Image Representations

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Introduction

Non-parametric Bayesian techniques are considered for learning dictionaries for sparse image representations, with applications in denoising, inpainting and compressive sensing. The four main contributions of our paper are:

- The dictionary is learned using a beta process construction, and therefore the number of dictionary elements and their relative importance may be inferred non-parametrically.
- For the denoising and inpainting applications, we do not have to assume a priori knowledge of the noise variance or sparsity level.
- The spatial inter-relationships between different components in images are exploited by use of the Dirichlet process and a probit stick-breaking process.
- Using learned dictionaries, inferred off-line or in situ, the proposed approach yields CS performance that is markedly better than existing standard CS methods as applied to imagery.

Model and Inference

Full likelihood of the Model

$$P(\mathbf{Y},\boldsymbol{\Sigma},\mathbf{D},\mathbf{Z},\mathbf{S},\boldsymbol{\pi},\gamma_s,\gamma_\epsilon)$$

$$=\prod_{i=1}^{N}\mathcal{N}(\boldsymbol{y}_i;\boldsymbol{\Sigma}_i\mathbf{D}(\boldsymbol{s}_i\odot\boldsymbol{z}_i),\gamma_\epsilon^{-1}\mathbf{I}_{\|\boldsymbol{\Sigma}_i\|_0})\mathcal{N}(\mathbf{s}_i;0,\gamma_s^{-1}\mathbf{I}_K)$$

$$\prod_{k=1}^{K}\mathcal{N}(\boldsymbol{d}_k;0,P^{-1}\mathbf{I}_P)Beta(\pi_k;a_0,b_0)$$

$$\prod_{i=1}^{N}\prod_{k=1}^{K}Bernoulli(z_{ik};\pi_k)$$

$$\Gamma(\gamma_s;c_0,d_0)\Gamma(\gamma_\epsilon;e_0,f_0)$$

Gibbs Sampling Inference

$$p(\boldsymbol{d}_k|-)\sim\mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{d}_k},\boldsymbol{\Sigma}_{\boldsymbol{d}_k})$$

$$\boldsymbol{\mu}_{\boldsymbol{d}_k}=\gamma_\epsilon\boldsymbol{\Sigma}_{\boldsymbol{d}_k}\sum_{i=1}^{N}z_{ik}s_{ik}\widetilde{\boldsymbol{x}}_i^{-k}\qquad \boldsymbol{\Sigma}_{\boldsymbol{d}_k}=\left(P\mathbf{I}+\gamma_\epsilon\sum_{i=1}^{N}z_{ik}^2s_{ik}^2\boldsymbol{\Sigma}_i^T\boldsymbol{\Sigma}_i\right)^{-1}$$

$$z_{ik}\sim Bernoulli\left(\frac{p_1}{p_0+p_1}\right)$$

$$p_1=\pi_k\exp\left[-\frac{\gamma_\epsilon}{2}(s_{ik}^2\boldsymbol{d}_k^T\boldsymbol{\Sigma}_i^T\boldsymbol{\Sigma}_i\boldsymbol{d}_k-2s_{ik}\boldsymbol{d}_k^T\widetilde{\boldsymbol{x}}_i^{-k})\right]\qquad p_0=1-\pi_k$$

$$p(s_{ik}|-)\sim\mathcal{N}(\mu_{s_{ik}},\Sigma_{s_{ik}})$$

$$\mu_{s_{ik}}=\gamma_\epsilon\Sigma_{s_{ik}}z_{ik}\mathbf{D}_k^T\boldsymbol{\Sigma}_i^T\boldsymbol{\Sigma}_i\widetilde{\mathbf{x}}_i^{-k}\qquad \Sigma_{s_{ik}}=\left(\gamma_s+\gamma_\epsilon z_{ik}^2\mathbf{D}_k^T\boldsymbol{\Sigma}_i^T\boldsymbol{\Sigma}_i\mathbf{D}_k\right)^{-1}$$

$$p(\pi_k|-)\sim Beta\left(\frac{a_0}{K}+\sum_{i=1}^{N}z_{ik},\frac{b_0(K-1)}{K}+N-\sum_{i=1}^{N}z_{ik}\right)$$

$$p(\gamma_s|-)\sim\Gamma\left(c_0+\frac{1}{2}KN,d_0+\frac{1}{2}\sum_{i=1}^{N}\mathbf{s}_i^T\mathbf{s}_i\right)$$

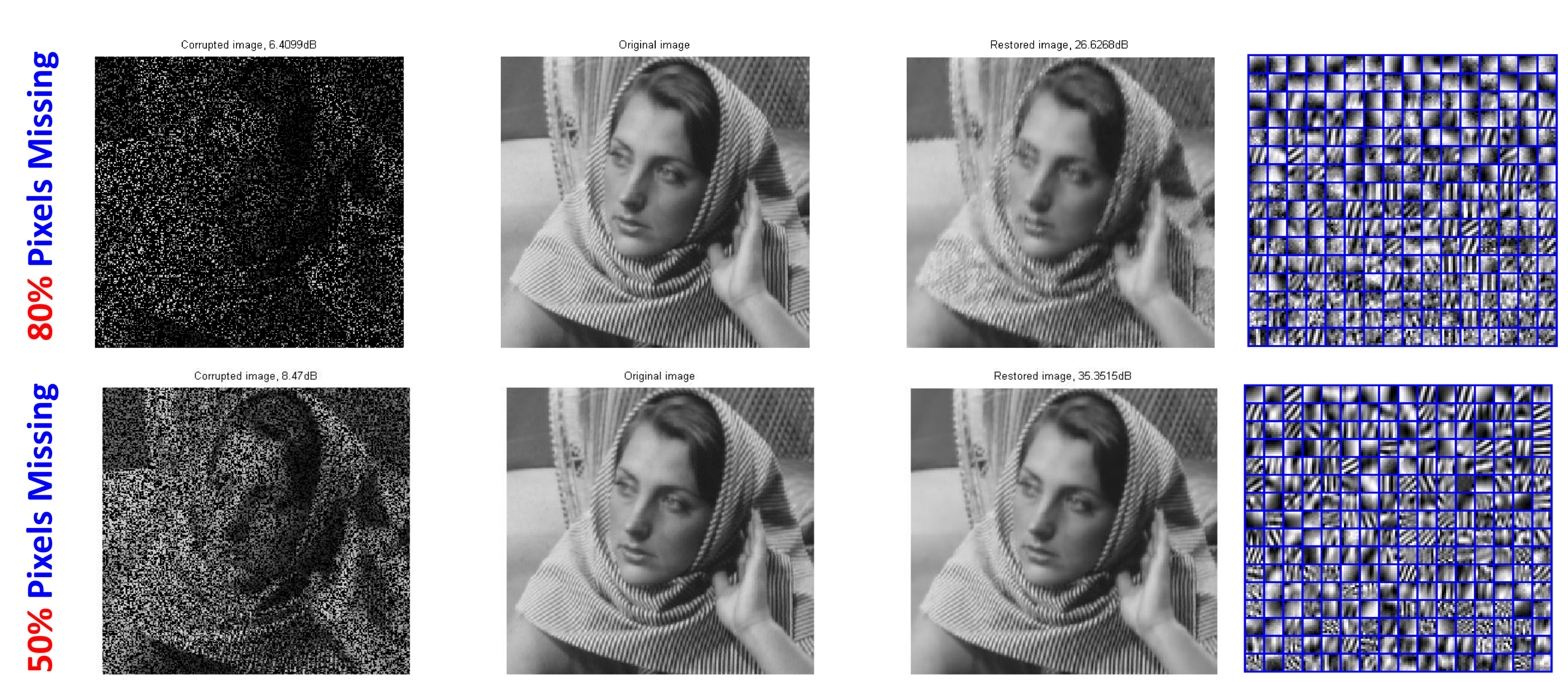
$$p(\gamma_\epsilon|-)\sim\Gamma\left(e_0+\frac{1}{2}\sum_{i=1}^{N}\|\boldsymbol{\Sigma}_i\|_0,f_0+\frac{1}{2}\sum_{i=1}^{N}\|\boldsymbol{\Sigma}_i^T\boldsymbol{y}_i-\boldsymbol{\Sigma}_i^T\boldsymbol{\Sigma}_i\mathbf{D}(\boldsymbol{s}_i\odot\boldsymbol{z}_i)\|_{\ell_2}\right)$$

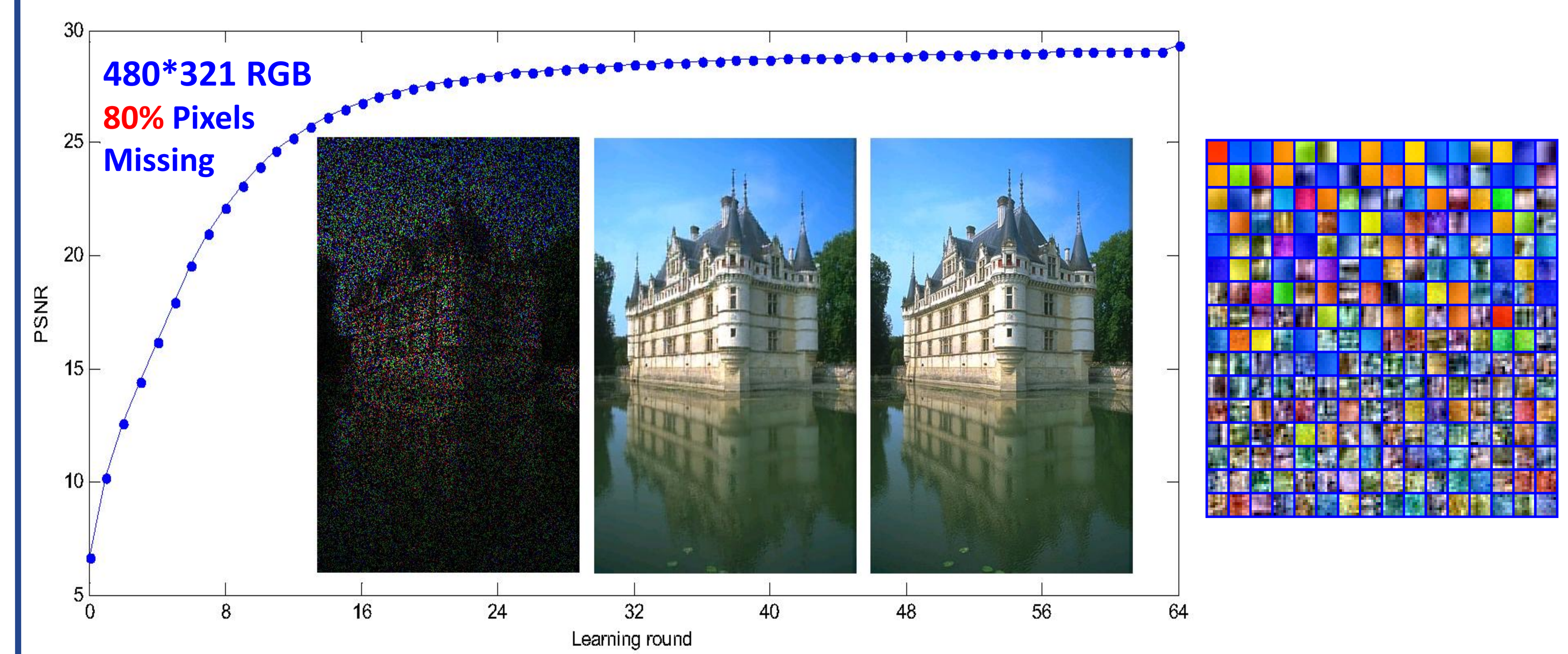
Applications

Image denoising

Original Noisy Image (dB)	K-SVD Denoising mismatched variance (dB)	K-SVD Denoising matched variance (dB)	Beta Process Denoising (dB)
24.58	30.67	34.32	34.52
20.19	31.52	32.15	32.19
14.56	19.60	27.95	27.95

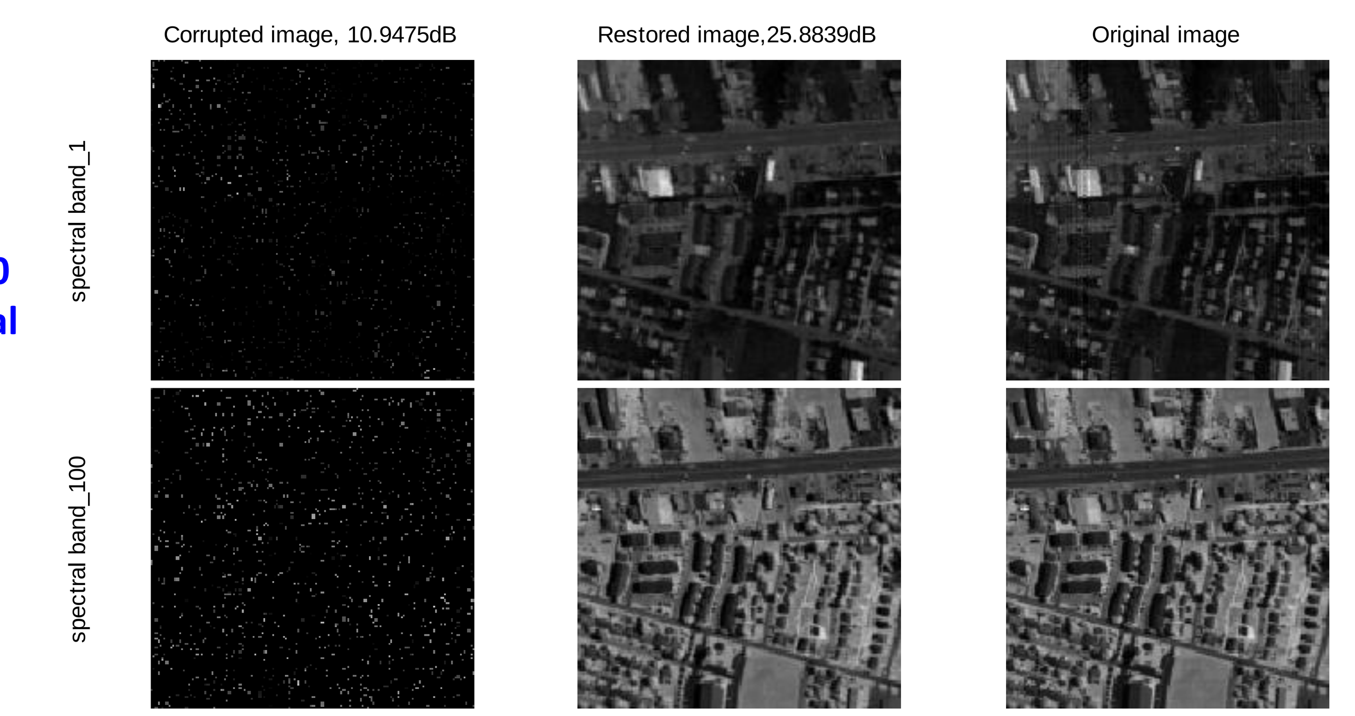
Image inpainting



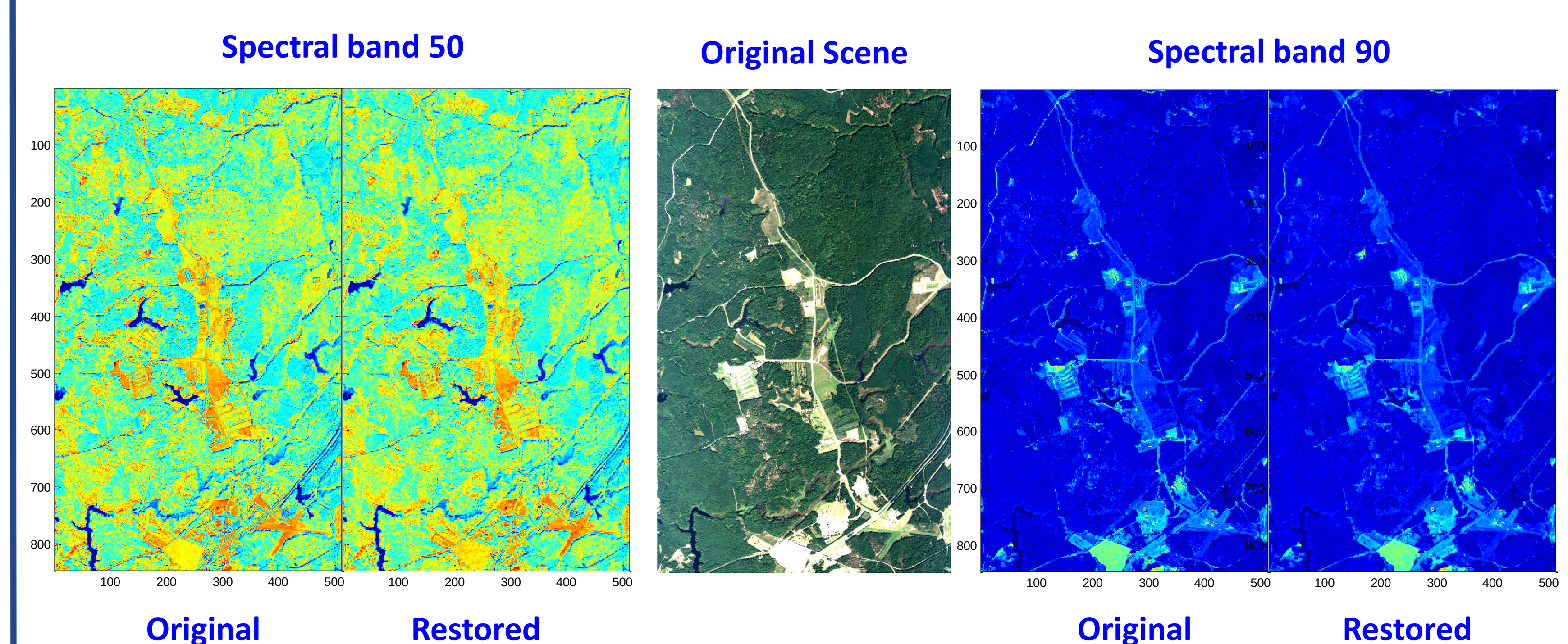


150*150*210 Hyperspectral

95% Pixels Missing

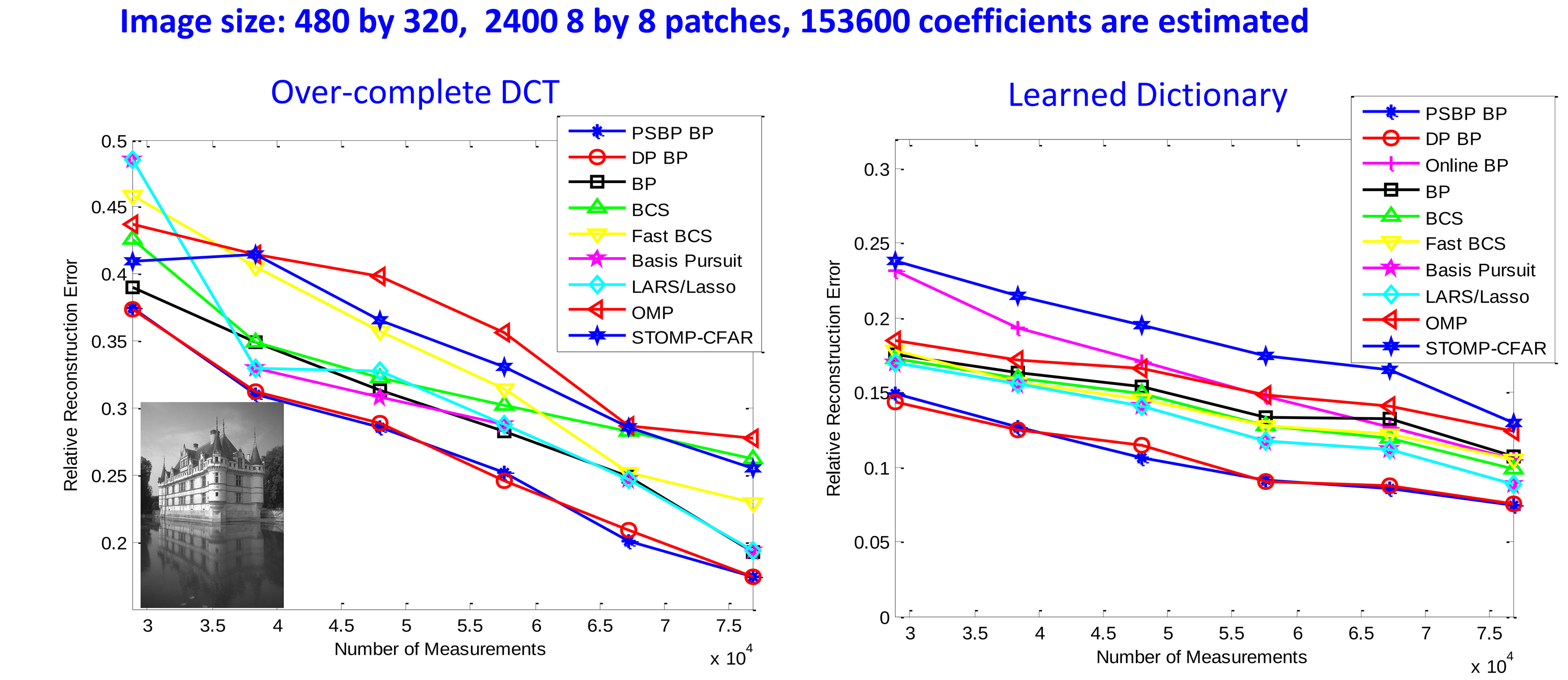


845*512*106 Hyperspectral, 98% pixels missing



Compressive sensing

Image size: 480 by 320, 2400 8 by 8 patches, 153600 coefficients are estimated



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