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Deep Latent Dirichlet Allocation with Topic-Layer-Adaptive Stochastic Gradient Riemannian MCMC

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Yulai Cong*, Bo Chen*, Hongwei Liu*, Mingyuan Zhou#

* National Laboratory of Radar Signal Processing,

* Collaborative Innovation Center of Information Sensing and Understanding,

Xidian University, Xi'an, Shaanxi, China

\# IROM Department, The University of Texas at Austin, Austin, TX, USA

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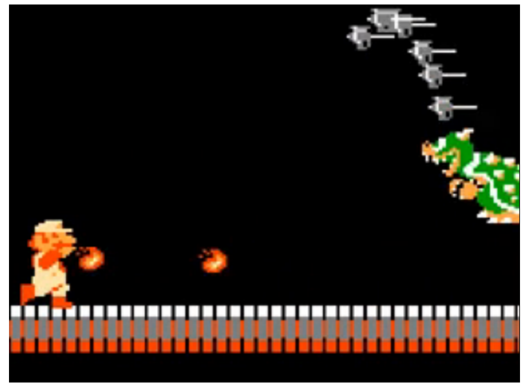
Outline

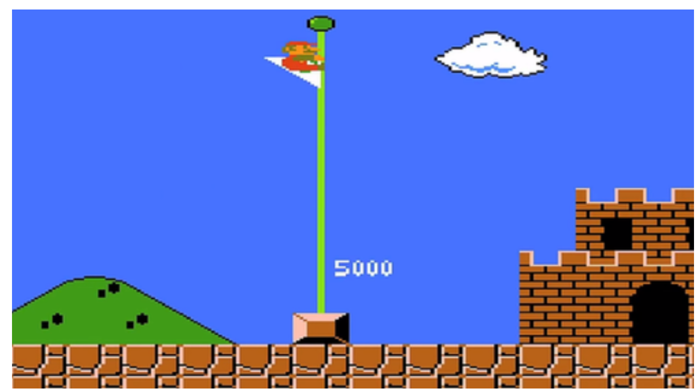


1.Motivation

3.Our Contribution

2. Barriers

4.Experiments



Motivation

Big data have

- ▶ Abundant information
- ▶ Huge volume
- ▶ …

Thus, we prefer

- ▶ Large-capacity models
  - Deep latent variable models (LVMs)
  - …
- ▶ Scalable inference methods
  - Stochastic Gradient (SG) MCMC
  - …

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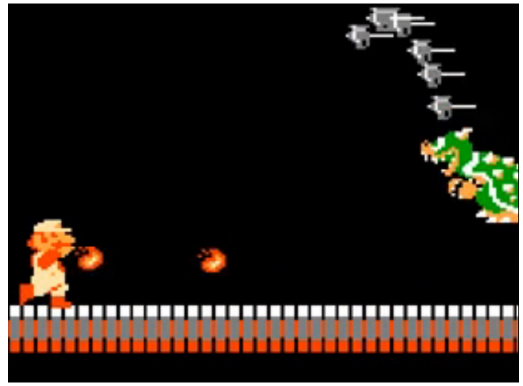
2. Barriers



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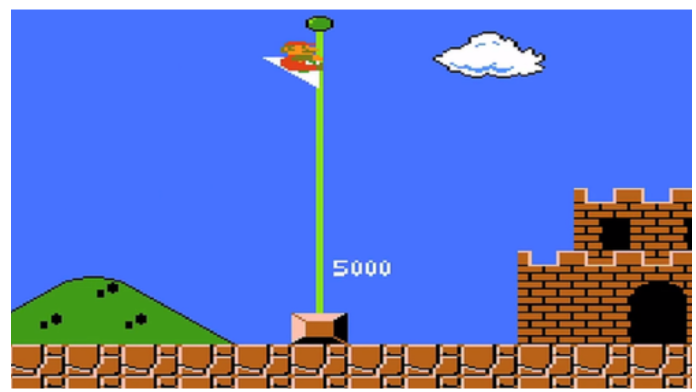
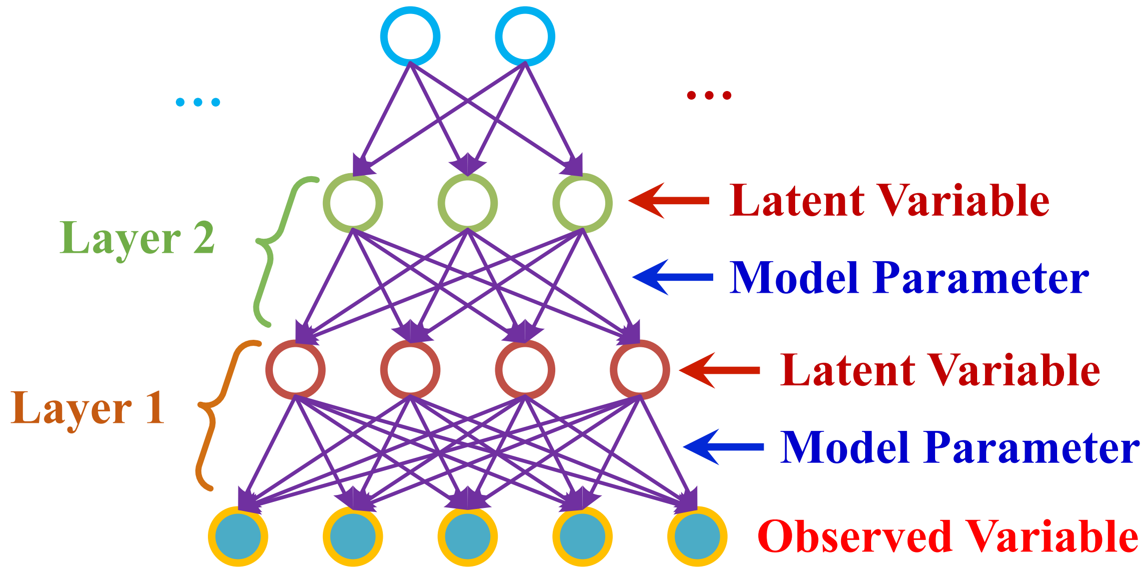


Barriers

To make a deep LVM scalable is challenging.

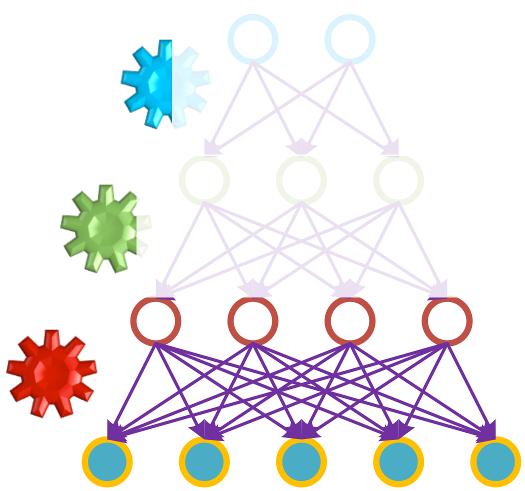


- ▶Gradients of model parameters are difficult to compute
- ▶Different layers, with different statistics, may require different learning rates

Barriers

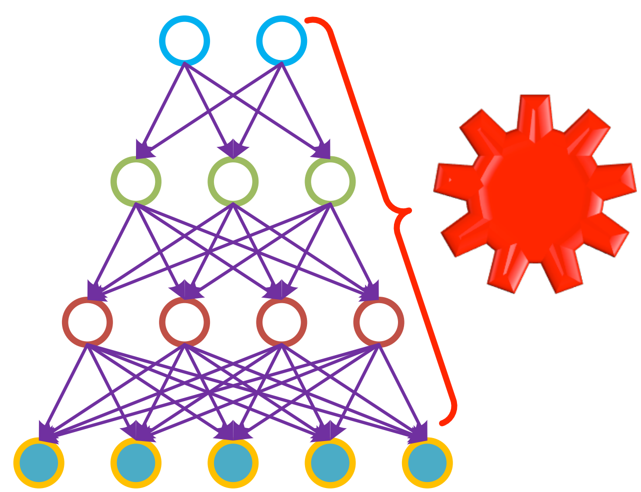
Most existing methods

- Scalable **but**
- Greedy layer-wise training
  - No communication between layers
- Shared SGD parameters



What we want

- Scalable **and**
- Principled joint learning
  - Communication & feedback between layers
- Adaptive SGD parameters



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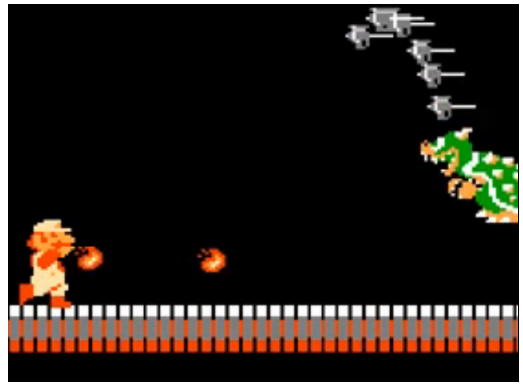

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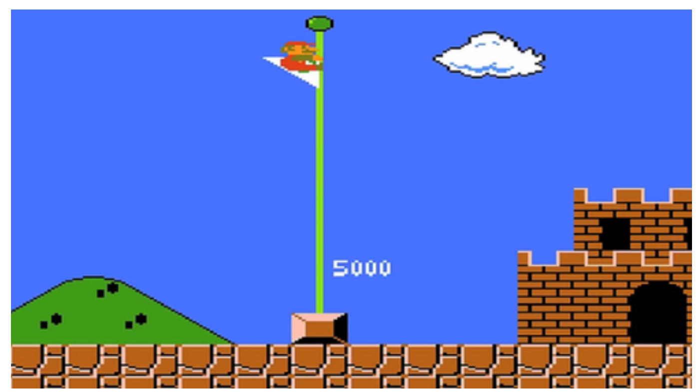
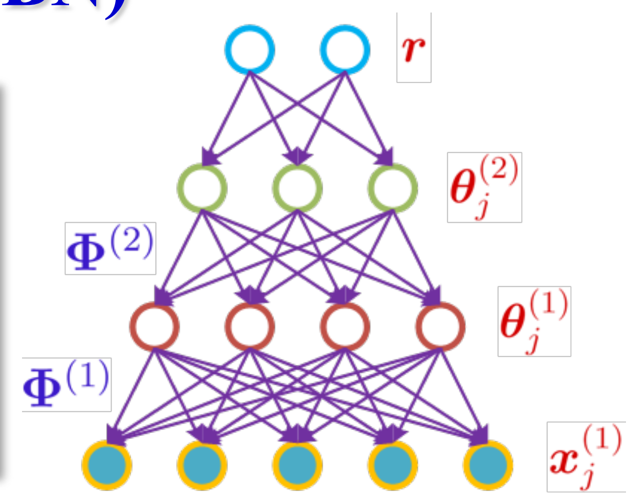


Our Contribution

We develop a principled joint SG-MCMC for a special deep LVM, namely the Poisson gamma belief network (PGBN).

❑ Poisson Gamma Belief Network (PGBN)

$$\boldsymbol{\theta}_j^{(L)} \sim \mathrm{Gam}\left(\boldsymbol{r}, 1/c_j^{(L+1)}\right),$$
$$\cdots$$
$$\boldsymbol{\theta}_j^{(l)} \sim \mathrm{Gam}\left(\boldsymbol{\Phi}^{(l+1)}\boldsymbol{\theta}_j^{(l+1)}, 1/c_j^{(l+1)}\right),$$
$$\cdots$$
$$\boldsymbol{x}_j^{(1)} \sim \mathrm{Pois}\left(\boldsymbol{\Phi}^{(1)}\boldsymbol{\theta}_j^{(1)}\right), \ \boldsymbol{\theta}_j^{(1)} \sim \mathrm{Gam}\left(\boldsymbol{\Phi}^{(2)}\boldsymbol{\theta}_j^{(2)}, \frac{p_j^{(2)}}{1-p_j^{(2)}}\right),$$



Priors: $\boxed{\phi_k^{(l)} \sim \mathrm{Dir}(\eta^{(l)}\mathbf{1}_{K_{l-1}})}$ $\quad \boldsymbol{r} \sim \mathrm{Gam}(\gamma_0/K_L, 1/c_0)$

$p_j^{(2)} \sim \mathrm{Beta}(a_0, b_0) \quad c_j^{(l)} \sim \mathrm{Gam}(e_0, 1/f_0)$

Gibbs

Not scalable

Preliminary for SG-MCMC

For the interested **batch posterior** $p(\boldsymbol{z}\,|X) \propto e^{-H(\boldsymbol{z})}$, one may get **posterior samples** using the **mini-batch** update rule as

$$\boldsymbol{z}_{t+1} = \boldsymbol{z}_t + \varepsilon_t \Big\{ -\left[\mathbf{D}(\boldsymbol{z}_t) + \mathbf{Q}(\boldsymbol{z}_t)\right] \nabla \tilde{H}(\boldsymbol{z}_t) + \Gamma(\boldsymbol{z}_t) \Big\} + \mathcal{N}\Big(\mathbf{0}, \varepsilon_t \left[2\mathbf{D}(\boldsymbol{z}_t) - \varepsilon_t \hat{\mathbf{B}}_t\right]\Big),$$

$$\nabla \tilde{H}(\boldsymbol{z}) = \nabla \left[-\ln p(\boldsymbol{z}) - \rho \sum\nolimits_{\boldsymbol{x} \in \tilde{X}} \ln p(\boldsymbol{x}\,|\boldsymbol{z})\right]$$

$\mathbf{D}(\boldsymbol{z})$: positive semi-definite; $\mathbf{Q}(\boldsymbol{z})$: skew-symmetric;

$$\Gamma_i(\boldsymbol{z}) = \sum\nolimits_j \frac{\partial}{\partial z_j}\left[\mathbf{D}_{ij}(\boldsymbol{z}) + \mathbf{Q}_{ij}(\boldsymbol{z})\right]$$

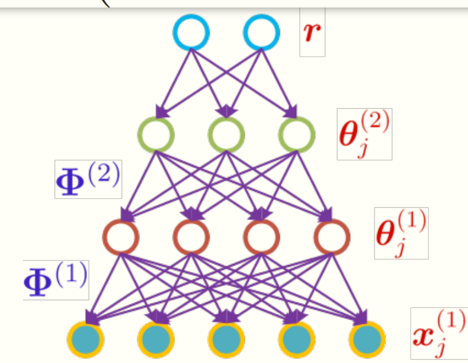
$\hat{\mathbf{B}}_t$: SG noise variance estimate.

SG-MCMC for PGBN

SG-MCMC & PGBN

- $\boldsymbol{z} = \left\{\boldsymbol{\Phi}^{(1)}, \cdots, \boldsymbol{\Phi}^{(L)}, \boldsymbol{r}\right\}$

$$\boldsymbol{z}_{t+1} = \boldsymbol{z}_t + \varepsilon_t \Big\{ -\big[\mathbf{D}(\boldsymbol{z}_t) + \mathbf{Q}(\boldsymbol{z}_t)\big] \nabla \tilde{H}(\boldsymbol{z}_t) + \Gamma(\boldsymbol{z}_t) \Big\} + \mathcal{N}\Big(\mathbf{0}, \varepsilon_t \big[2\mathbf{D}(\boldsymbol{z}_t) - \varepsilon_t \hat{\mathbf{B}}_t\big]\Big),$$



For simplicity, we use

- $\mathbf{Q}(\boldsymbol{z}) = \mathbf{0}, \hat{\mathbf{B}}_t = \mathbf{0}$
- $\mathbf{D}(\boldsymbol{z}) = \mathbf{G}(\boldsymbol{z})^{-1}$
  - $\mathbf{G}(\boldsymbol{z})$: Fisher information matrix (FIM)

$$\boldsymbol{z}_{t+1} = \boldsymbol{z}_t + \varepsilon_t \left\{ -\mathbf{G}(\boldsymbol{z}_t)^{-1} \nabla \tilde{H}(\boldsymbol{z}_t) + \Gamma(\boldsymbol{z}_t) \right\} + \mathcal{N}\left(\mathbf{0}, 2\varepsilon_t \mathbf{G}(\boldsymbol{z}_t)^{-1}\right)$$

We need

- $\mathbf{G}(\boldsymbol{z})^{-1}$
- $\nabla \tilde{H}(\boldsymbol{z})$

However

- $\mathbf{G}(\boldsymbol{z})$ of PGBN is intractable
- $\mathbf{G}(\boldsymbol{z})^{-1}$ may be expensive

Calculate FIM

What we find: An alternative representation of the PGBN makes it straightforward to calculate $\mathbf{G}(\boldsymbol{z})$.

Deep Latent Dirichlet Allocation (DLDA)

Techniques:

- Data augmentation
- Marginalization

- $q_j^{(1)} := 1$
- $q_j^{(l+1)} = \ln\left(1 + q_j^{(l)}/c_j^{(l+1)}\right)$
- $p_j^{(l)} := 1 - e^{-q_j^{(l)}}$
- $\tilde{p} := q_\cdot^{(L+1)}/(c_0 + q_\cdot^{(L+1)})$

$$x_{k\cdot}^{(L+1)} \sim \mathrm{Log}(\tilde{p}), K_L \sim \mathrm{Pois}[-\gamma_0 \ln(1-\tilde{p})],$$
$$X^{(L+1)} = \textstyle\sum_{k=1}^{K_L} x_{k\cdot}^{(L+1)} \delta_{\phi_k^{(L)}},$$
$$\left(x_{vj}^{(L+1)}\right)_j \sim \mathrm{Mult}\left[x_{v\cdot}^{(L+1)}, \left(q_j^{(L+1)}\right)_j / q_\cdot^{(L+1)}\right],$$
$$m_{vj}^{(L)(L+1)} \sim \mathrm{SumLog}(x_{vj}^{(L+1)}, p_j^{(L+1)}),$$
$$\cdots$$
$$x_{vj}^{(l)} = \textstyle\sum_{k=1}^{K_l} x_{vkj}^{(l)}, \left(x_{vkj}^{(l)}\right)_v \sim \mathrm{Mult}\left(m_{kj}^{(l)(l+1)}, \phi_k^{(l)}\right),$$
$$m_{vj}^{(l-1)(l)} \sim \mathrm{SumLog}(x_{vj}^{(l)}, p_j^{(l)}),$$
$$\cdots$$
$$x_{vj}^{(1)} = \textstyle\sum_{k=1}^{K_1} x_{vkj}^{(1)}, \left(x_{vkj}^{(1)}\right)_v \sim \mathrm{Mult}\left(m_{kj}^{(1)(2)}, \phi_k^{(1)}\right).$$

Calculate FIM

The alternative DLDA representation ➔

A *block-diagonal* and thus *easily inverted* FIM as

$$\mathbf{G}(\boldsymbol{z}) = \mathrm{diag}\left[\mathbf{I}\left(\phi_1^{(1)}\right), \cdots, \mathbf{I}\left(\phi_{K_L}^{(L)}\right), \mathbf{I}(\boldsymbol{r})\right]$$

Properties

- **Analytical & Practical**
- **Similar to the Hessian matrix in optimization**
- **Principled joint inference of the PGBN (DLDA)**
- **A separate puzzle for each $\phi_k^{(l)}$ and $\boldsymbol{r}$**

$$\boldsymbol{z}_{t+1} = \boldsymbol{z}_t + \varepsilon_t \left\{-\mathbf{G}(\boldsymbol{z}_t)^{-1}\nabla\tilde{H}(\boldsymbol{z}_t) + \Gamma(\boldsymbol{z}_t)\right\} + \mathcal{N}\left(\mathbf{0}, 2\varepsilon_t\mathbf{G}(\boldsymbol{z}_t)^{-1}\right)$$

Efficient Inference for Topics $\phi_k^{(l)}$

Reduced-Mean Inference on the Probability Simplex

For the batch posterior $(\phi_k|-) \sim \mathrm{Dir}(x_{1k\cdot}+\eta, \ldots, x_{Vk\cdot}+\eta)$ on the probability simplex, we derive a *mini-batch* update rule as

$$(\phi_k)_{t+1} = \left[(\phi_k)_t + \frac{\varepsilon_t}{M_k} \left[(\rho \tilde{\boldsymbol{x}}_{:k\cdot} + \eta) - (\rho \tilde{x}_{\cdot k\cdot} + \eta V)(\phi_k)_t \right] + \mathcal{N}\left(\mathbf{0}, \frac{2\varepsilon_t}{M_k} \mathrm{diag}\,(\phi_k)_t\right) \right]_\angle$$

where $\tilde{x}_{vk\cdot}$ comes from mini-batches.

Techniques

- **Reduced-mean parameterization φ_k of ϕ_k, namely**

$$\phi_k = \left((\varphi_k)^T, 1 - \sum\nolimits_v \varphi_{vk}\right)^T$$

- **Fast simulation method in [1]**

[1] Cong, Y., Chen, B., and Zhou, M. Fast simulation of hyperplane-truncated multivariate normal distributions. Bayesian Analysis Advance Publication, 2017.

The Whole Picture

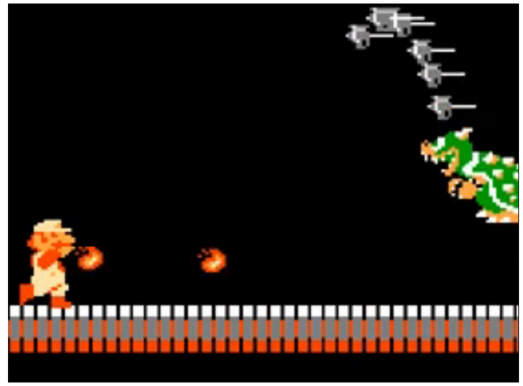
- **Topic-Layer-Adaptive Stochastic Gradient Riemannian (TLASGR) MCMC**

Algorithm 1 TLASGR MCMC for DLDA (PGBN).

Input: Data mini-batches;
Output: Global parameters of DLDA (PGBN).

for $t = 1, 2, \cdots$ **do**
    / Collect local information*
    Upward-downward Gibbs sampling (Zhou et al., 2016a) on the t^{th} mini-batch for $\tilde{\boldsymbol{x}}_{:k\cdot\cdot}$, $\tilde{x}_{\cdot k\cdot\cdot}$, $\tilde{\boldsymbol{x}}_{:\cdot}^{(L+1)}$, and $\tilde{q}_{\cdot}^{(L+1)}$;
    / Update global parameters*
    for $l = 1, \cdots, L$ and $k = 1, \cdots, K_l$ **do**
        Update $M_k^{(l)}$ with (18); then $\boldsymbol{\phi}_k^{(l)}$ with (15);
    end for
    Update $M^{(L+1)}$ with (19) and then $\boldsymbol{r}$ with (17).
end for

Outline



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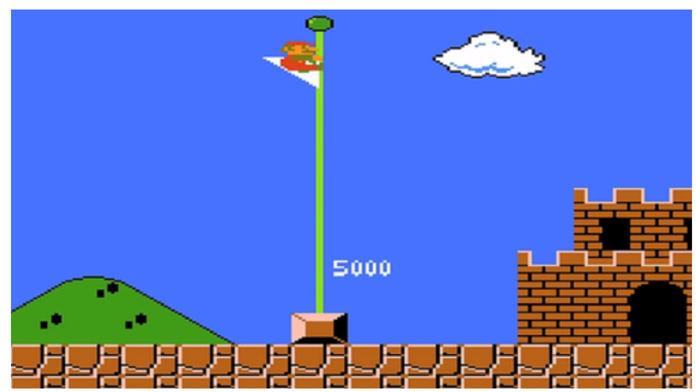


Experiments

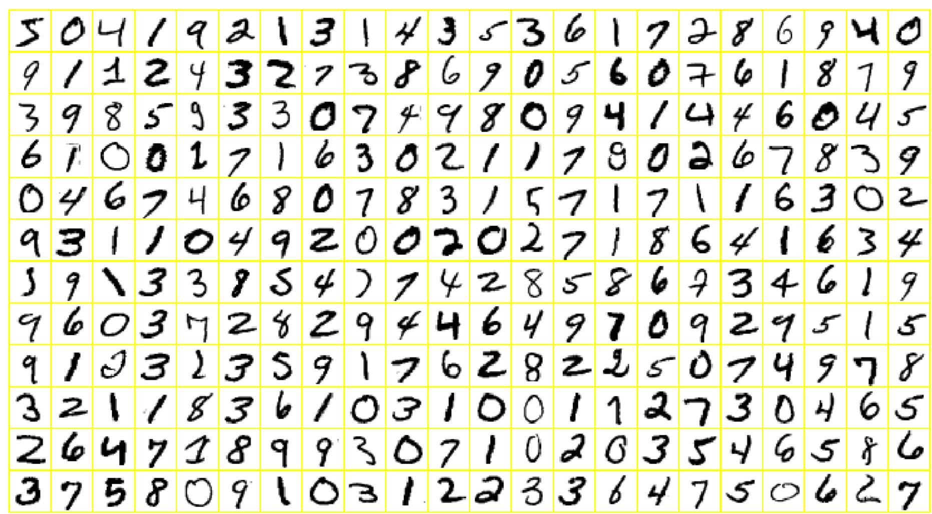
Three benchmark corpora

- ▶20Newsgroups (20News)
- ▶Reuters Corpus Volume I (RCV1)
- ▶Wikipedia (Wiki)

	20News	RCV1	**Wiki**
Vocabulary	2,000	10,000	**7,702**
Training	11,315	794,414	**≈10M**
Test	7,531	10,000	**1,000**

MNIST digits

- ▶**For illustration**
- ▶Size: 28×28
- ▶Training: 60000
- ▶Test: 10000



Performance: Perplexity

- ▶ Smaller is better
- ▶ Bold: Top two

DLDA & Gibbs

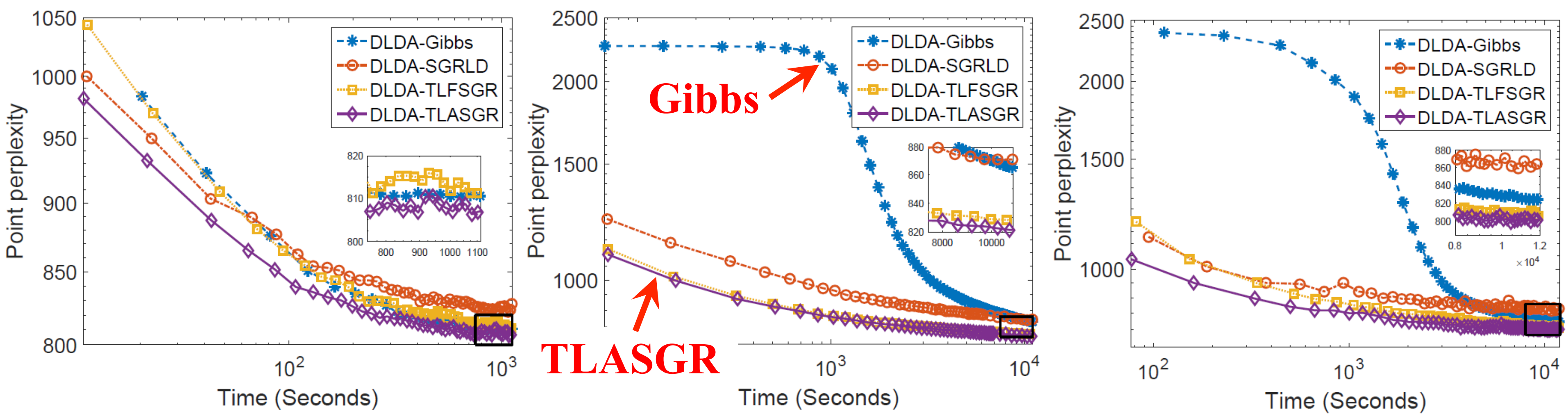
- ▶ Best results
- ▶ Not scalable

DLDA & TLASGR

- ▶ Second best
  - Comparable to Gibbs
- ▶ Scalable

Model	Method	Size	20 News	RCV1	Wiki
DLDA	TLASGR	128-64-32	757	815	**786**
DLDA	TLASGR	128-64	758	817	787
DLDA	TLASGR	128	770	823	802
DLDA	TLFSGR	128-64-32	760	817	789
DLDA	TLFSGR	128-64	759	819	791
DLDA	TLFSGR	128	772	829	804
DLDA	SGRLD	128-64-32	775	827	792
DLDA	SGRLD	128-64	770	823	792
DLDA	SGRLD	128	777	829	803
DLDA	Gibbs	128-64-32	**752**	**802**	—
DLDA	Gibbs	128-64	**754**	**804**	—
DLDA	Gibbs	128	768	818	—
DPFM	SVI	128-64	818	961	791
DPFM	MCMC	128-64	780	908	**783**
DPFA-SBN	SGNHT	128-64-32	827	1143	876
DPFA-RBM	SGNHT	128-64-32	896	920	942
nHDP	SVI	(10,10,5)	889	1041	932
LDA	Gibbs	128	893	1179	1059
FTM	Gibbs	128	887	1155	991
RSM	CD5	128	877	1171	1001

Performance: Scalability



(a) A single-layer DLDA on 20News (b) DLDA of size 128-64 on RCV1 (c) DLDA of size 128-64 on Wiki

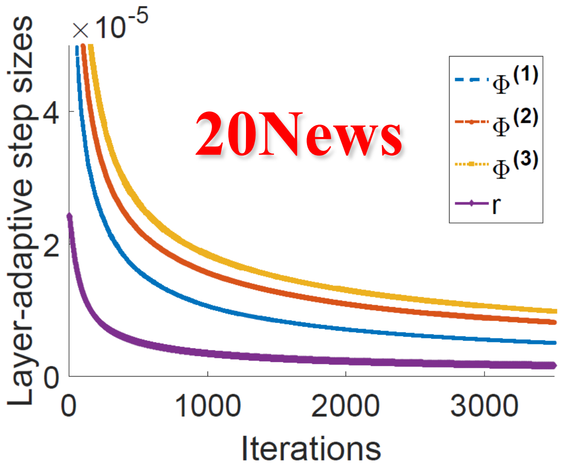
20News Train: **11K**

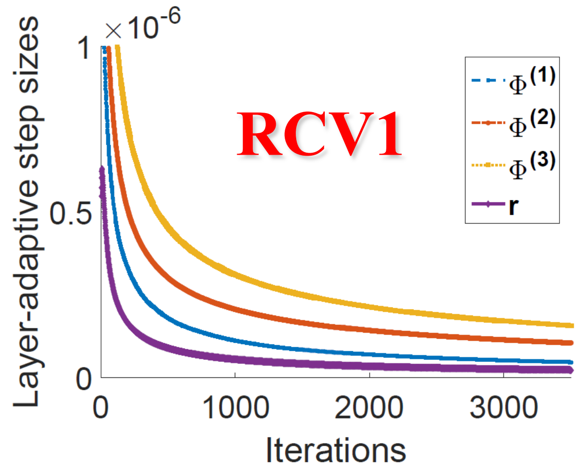
RCV1 Train: **0.8M**

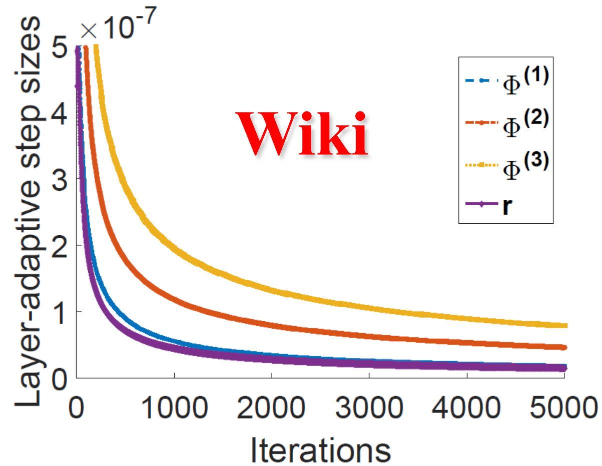
Wiki Train: **10M**

Benefits of Joint Learning

▶Topic-layer-adaptive learning rates





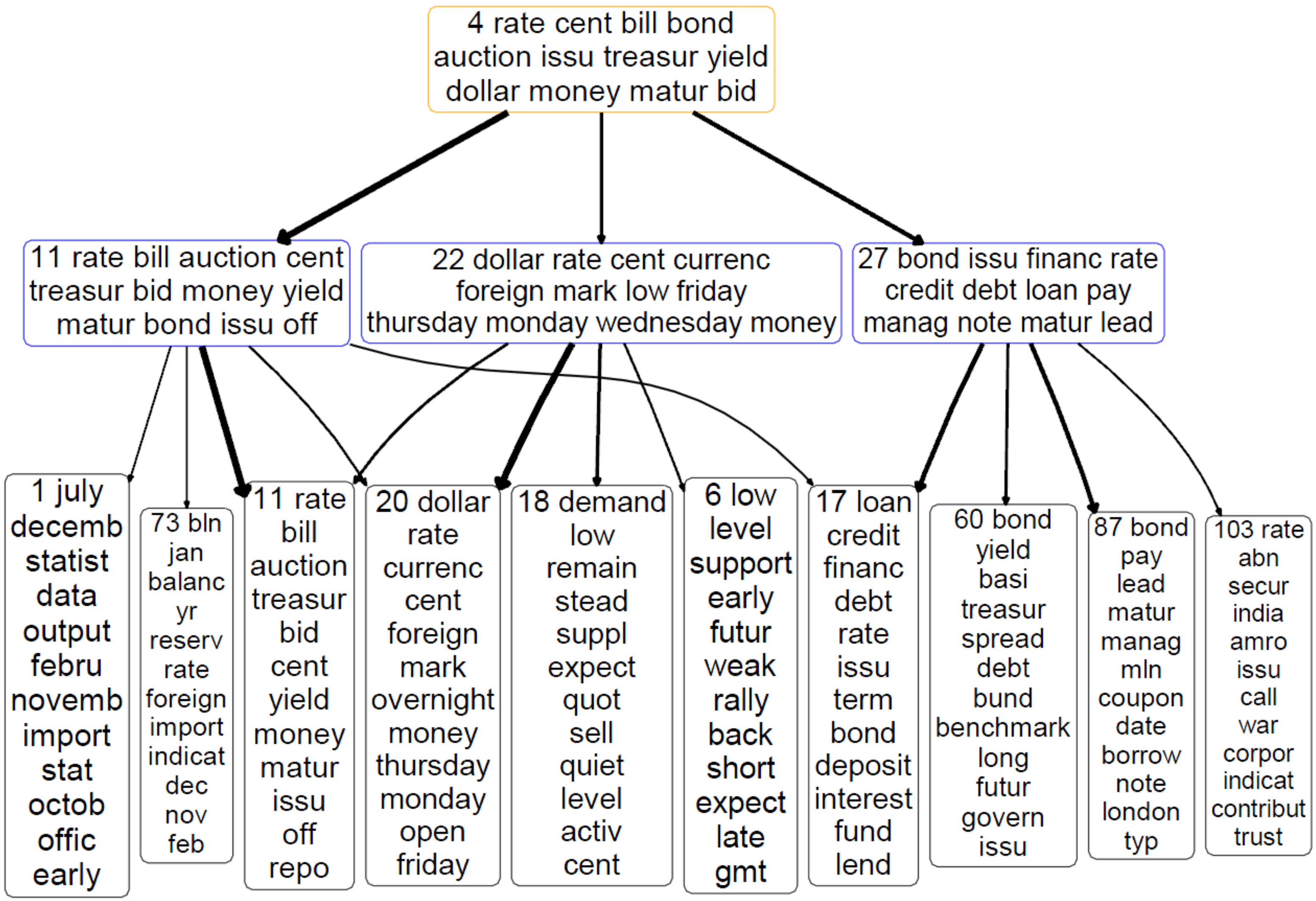


▶Better information propagation

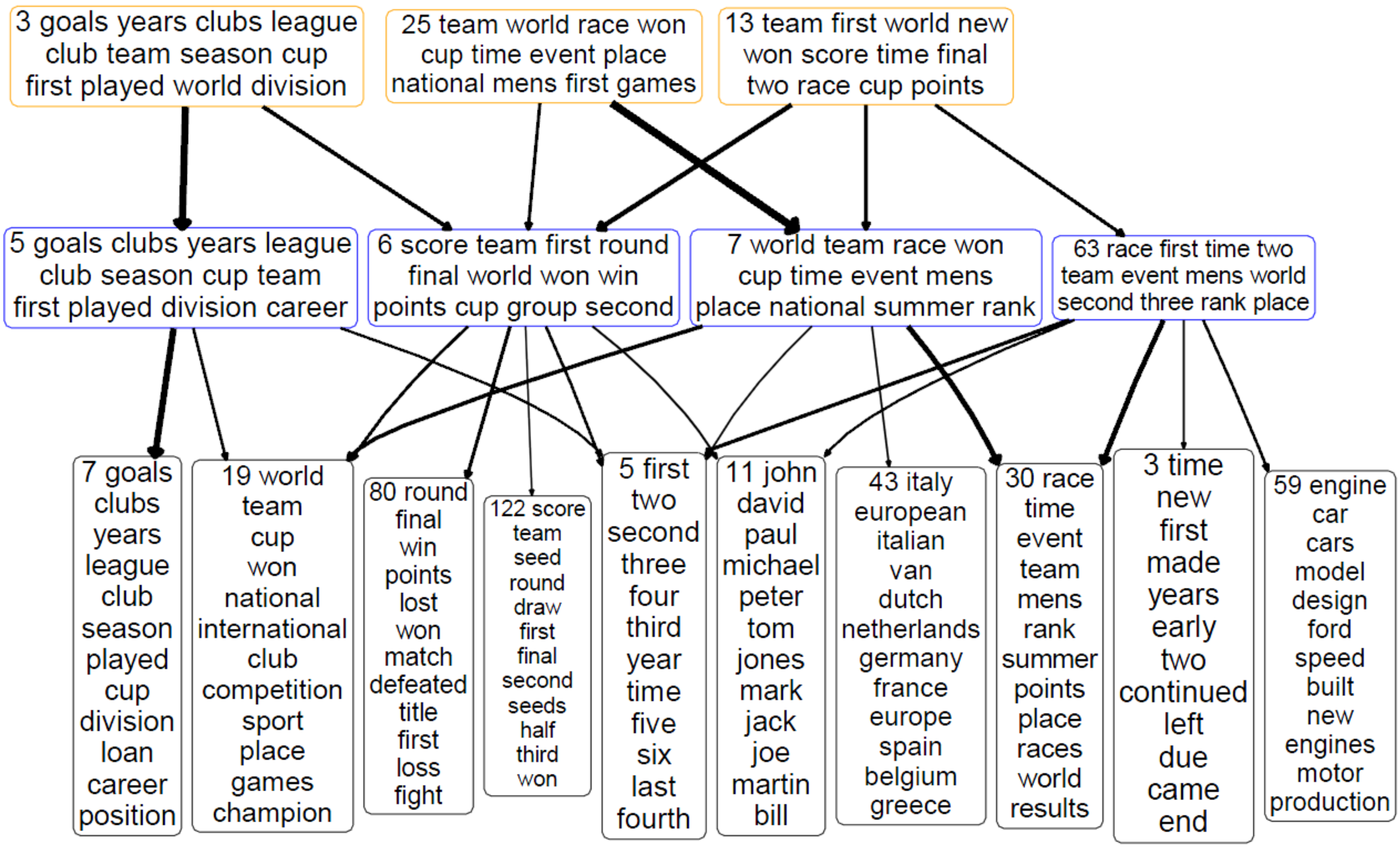
Topic-layer-FIXED learning rates

Topic-layer-ADAPTIVE learning rates

Example Topics: RCV1



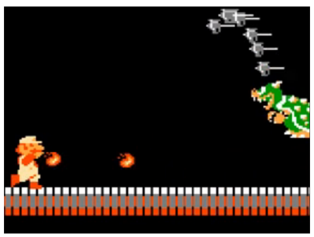
Example Topics: Wiki



Conclusions

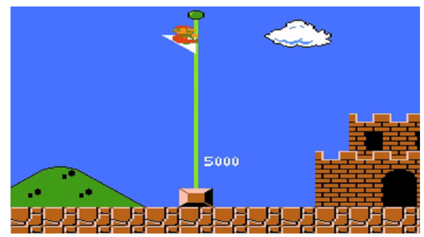
Scalable inference of deep LVMs for big data

- Gradients are difficult to calculate
- Joint learning of all layers of deep models



For the PGBN, we develop the principled joint TLASGR via newly introduced

- An alternative representation DLDA
- Analytical and block-diagonal FIM
- Reduced-mean parameterization on probability simplex



Experimentally, TLASGR

- Performance comparable to Gibbs
- Scalable to huge data
- Principle joint learning
  - Topic layer adaptive learning rates
  - Better information propagation
- Interpretable topics

THANK YOU!