The integer-valued beta-negative binomial process (BNBP) is employed to partition a count vector into a latent column-exchangeable random count matrix. The paper makes the following contributions:

- An exchangeable partition probability function (EPPF) for mixed-membership modeling is proposed.
- The prediction rule to simulate the exchangeable random partitions for grouped data governed by the BNBP is derived.
- A fully collapsed Gibbs sampler, with closed-form update equations for model parameters, is constructed.
- The BNBP topic model is shown to converge fast, mix well, and provide state-of-the-art prediction performance when a compact representation of the corpus is desired.

**Model and Inference**

- **Beta Negative Binomial Process**
  \[ X_j \sim NBP(r_j, B), \quad B \sim \text{BP}(c, B_0) \]

  Augmented representation: \( X_j | \Theta_j \sim \text{PP}(\Theta_j), \quad \Theta_j | r_j, B \sim \text{GP}[r_j, B/(1-B)], \quad B \sim \text{BP}(c, B_0) \)

- **Group size dependent mixture model**
  \[ z_{ji} \sim \sum_{k=1}^{\infty} \frac{\delta_{\tau k}}{\delta_{\tau k}} \delta_k, \quad m_j \sim \text{Pois}(\Theta_j(\Omega)), \quad \Theta_j \sim \text{GP}[r_j, B/(1-B)], \quad B \sim \text{BP}(c, B_0) \]

  Conditional Likelihood:
  \[ f(z, m | r, c, \gamma_0, c) \propto \prod_{k=1}^{\infty} \frac{\Gamma(n_{jk}+r_j)}{\Gamma(r_j)} p_k^{n_{jk}} (1-p_k)^{r_j} \]

**Mixed-Membership Modeling**

- **BNBP Topic Model**
  \[ x_{ji} \sim \text{Multi}(\phi_{zi}), \quad \phi_j \sim \text{Dir}(\eta, \ldots, \eta), \quad z_{ji} \sim \sum_{k=1}^{\infty} \frac{\delta_{\tau k}}{\delta_{\tau k}} \delta_k, \quad m_j \sim \text{Pois}(\Theta_j(\Omega)), \quad \Theta_j \sim \text{GP}[r_j, B/(1-B)], \quad B \sim \text{BP}(c, B_0), \gamma_0 \sim \text{Gamma}(e_0, 1/f_0) \]

  **Parameter Inference**
  \[ (\gamma_0 | -) \sim \text{Gamma}(e_0 + K_j, \gamma_0 + \psi(c+c_r) - \psi(c)), \quad (p_k | -) \sim \text{Beta}(n_{jk}, c + r_j), \quad \text{Q}(\Omega|D_j | -) \sim \log \text{Beta}(\gamma_0, c + r_j) \]

  **HDP-LDA Collapsed Gibbs Sampler**
  \[ P(z_{ji} = k | \alpha, \gamma_0, c, r) \propto \frac{\Gamma(n_{jk}+r_j)}{\Gamma(r_j)} p_k^{n_{jk}} (1-p_k)^{r_j} \]

  **BNBP Collapsed Gibbs Sampler**
  \[ P(z_{ji} = k | \alpha, \gamma_0, c, r) \propto \frac{\Gamma(n_{jk}+r_j)}{\Gamma(r_j)} p_k^{n_{jk}} (1-p_k)^{r_j} \]